calculated for a cesium plasma on the basis of a similar model [20]. The transition curves show a course that is comparable with the one given in Figures 2 and 3.

Finally, comparing our results with the solutions of numerical calculations [3] we find in the limiting cases of optical thin and thick plasmas an agreement within about 50%. The transition curves between the two limiting cases may have a greater inaccuracy, say, of about a factor of two.

The author is grateful to Professor C. M. BRAAMS for his stimulating interest and to Dr. A. J. POSTMA for critically reading the manuscript.

This work was performed as part of the research programme of the association agreement of Euratom and the "Stichting voor Fundamenteel Onderzoek der Materie" (FOM) with financial support from the "Nederlandse Organisatie voor Zuiver-Wetenschappelijk Onderzoek" (ZWO) and Euratom.

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On the Heating of a Pinch

R. MEWE *

Association Euratom-FOM, FOM-Instituut voor Plasma-Fysica
Rijnhuizen, Jutphaas, The Netherlands

(Z. Naturforsch. 25 a, 1803—1807 [1970]; received 24 August 1970)

The compression temperature of a theta pinch is calculated as a function of the circuit parameters and the final β-value of the plasma. One of the results is that the temperature, $\hat{T}$, at the peak magnetic field, $\hat{B}$, scales of $(\dot{B}\hat{B})^{2/3}$, where $\dot{B}$ is the initial rate of rise of the magnetic field. A possibility of combining two capacitor banks to increase the implosion heating rate is discussed.

1. Introduction

In the course of work on screw pinches at Jutphaas it has been shown [1–5] that for a toroidal screw pinch with a longitudinal current of the order of the Kruskal-Shafranov limit to be stable against kink modes the β of the plasma must be kept low, say, $\beta \lesssim 0.25$ (β = kinetic plasma pressure/confining magnetic field pressure). In such a screw pinch the ratio of the axial and azimuthal magnetic field is of the order of R/r_1, in which R is the major and r_1 is the minor radius of the toroidal discharge tube [1–4]. As in practice $R/r_1 \gg 1$, the plasma compression is mainly determined by the axial field, so that the heating mechanism is similar to that in a common theta pinch.

The theta pinch is well known for its capability to produce a hot and dense plasma with a high β. However, if the β is lowered by trapping a magnetic bias field parallel to the main confining field, the heating becomes less effective because of the reduction of the compression ratio [6–8]. The production of a low-β pinch — such as the above-mentioned screw pinch — therefore requires a large capacitor bank. This paper gives the formulae necessary to calculate the parameters of this bank. A reduction of costs is possible if, in addition to the main capacitor bank, a small fast bank is used. This method, described earlier by DE VRIES [3], has the advantage of a high initial field rise and therefore of an increased heating during the fast implosion.

2. Formulae for the Heating of a Theta Pinch

In a theta pinch device the plasma is produced and heated by the shock and adiabatic compression due to a fast rising magnetic field induced by discharging a low-inductance capacitor bank through a coil around the discharge tube.

* Reprints request to the present address: R. MEWE, Laboratorium voor Ruimteonderzoek, University of Utrecht, *Utrecht*, The Netherlands.

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The plasma temperature at the end of the irreversible implosion phase, during which primarily the ions are heated, is computed to be [9]

$$k\,T_s = \tfrac{2}{9}\,a\,b\,r_1\,\dot{B}\left\{\frac{m_i}{n_0\,\mu_0}\right\}^{1/2}. \qquad (1)$$

This is the implosion temperature after the ions and electrons have sheared their energies, but the result of the end temperature in Eq. (4) remains the same if equipartition is achieved later during the adiabatic compression. The index s refers to the end of the shock. Further, n_0 is the initial density of the ions and atoms (after dissociation in the case of a molecular gas), m_i is the ion mass, and $\dot{B}$ is the time derivative of the initial magnetic field if no plasma were present in the tube. The numerical factor a depends on the model used in the calculations, e.g., $a = 1$ (free-particle model), $a \approx 1/3$ (snow-plow model)[9]; taking $a = 0.6$ we obtain a good agreement with measurements of the ion energy in fast theta pinches in deuterium[10, 11]. Finally, the factor b indicates how the energy transferred from the electrical circuit to the plasma depends on the plasma radius, r_s, at the end of the shock and on the circuit parameter λ (cf. Fig. 1). Here λ is the ratio, L_i/L_e, of the internal inductance (within the discharge tube) and the external inductance (of capacitor bank, collector, cables, and the space between the inner tube wall and the coil).

After the fast implosion the plasma is further heated by the reversible adiabatic compression due to the still rising magnetic field. A simple model for the adiabatic compression of a homogeneous plasma without energy and particle losses yields that the plasma temperature scales with the magnetic field as $T \propto B^{4/(6-\beta)}$ (ratio of specific heats $\gamma = 5/3$)[12]; for $\beta \approx 1$ this reduces to $T \propto B^{4/5}$. This dependence has been confirmed, e.g., by soft X-ray measurements of the electron temperature[13]. In the case $\beta \ll 1$, in which we are mainly interested, we have $T \propto B^{2/3}$. Since during the adiabatic compression β varies with B[14], it is not possible to give an exact explicit expression for the whole range of β-values, but to a good approximation we can write[15]

$$\hat{T}/T_s \approx (\hat{B}/B_s)^{(2+0.4\hat{\beta})/3} \approx 1.45^{\hat{\beta}}(\hat{B}/B_s)^{2/3}, \qquad (2)$$

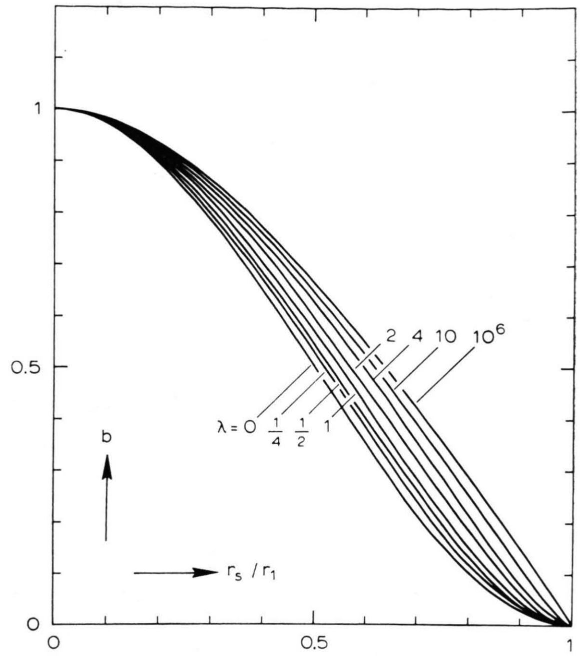

Fig. 1. Reduction of implosion energy as a function of the plasma radius.

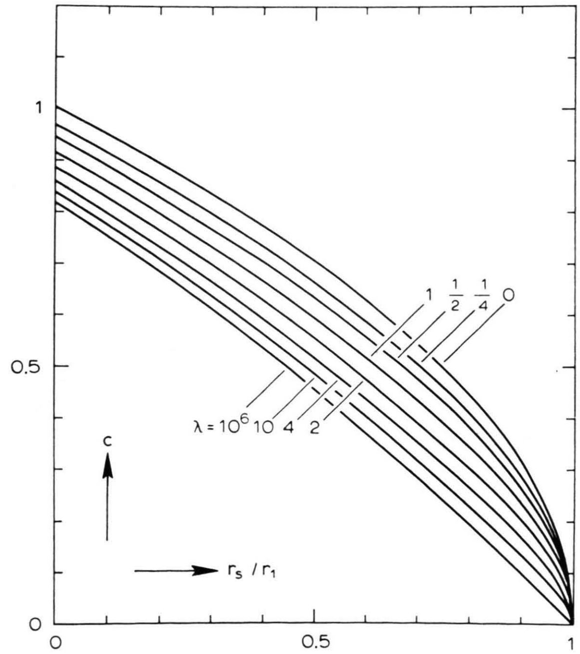

Fig. 2. Reduction of implosion time as a function of the plasma radius.

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where B_s is the magnetic field after the shock. The symbol ^ refers to the end of the adiabatic phase, i.e., to the moment, $\hat{t}$, where B has its maximum value, $\hat{B}$. Thus $\hat{\beta}$ is the value of β at the moment $\hat{t}$. As in practice $(\hat{B}/B_s)^{2/15}$ varies very slowly, i.e., between about 1.3 and 1.6, we have taken $(\hat{B}/B_s)^{2/15} = 1.45$ as a practical mean value. The field B_s is given by

$$B_s \approx \dot{B}\, t_s \approx 2\, c\{r_1 \dot{B} (n_0\, m_i\, \mu_0)^{1/2}\}^{1/2}, \qquad (3)$$

where t_s is the characteristic implosion time, and c is a numerical reduction factor (cf. Fig. 2)[9]. In the derivation of Eqs. (1) and (3) it was assumed that the initial field rises approximately linearly with time. Combining Eqs. (1), (2), and (3) we obtain

$$k\hat{T} = \frac{2^{3/3}}{9}\, a\, d \left\{\frac{r_1 \dot{B} \hat{B}\, m_i^{1/2}}{n_0\, \mu_0}\right\}^{2/3}, \qquad (4)$$

with $d = 1.45^{\hat{\beta}}\, b\, c^{-2/3}$, or inserting numerical values for deuterium and taking $a = 0.6$

$$k\hat{T} = 200\, d\, (r_1 \dot{B} \hat{B}/p)^{2/3}, \qquad (5)$$

where the maximum plasma temperature $k\hat{T}$ is expressed in eV, the inner tube radius r_1 in cm, the field rise $\dot{B}$ in Tesla/μs, the field amplitude $\hat{B}$ in Tesla, and the filling pressure p in mTorr. Because r_s can be related to $\hat{\beta}$ through the pressure balance and through the formulae for the adiabatic compression, the factor d can be computed[15] as a function of $\hat{\beta}$ and λ; the results are shown in Fig. 3. This factor indicates the relative heating efficiency as a function of $\hat{\beta}$ and λ. The effects of the fast and the adiabatic compression are represented by the factors $b\, c^{-2/3}$ and $1.45^{\hat{\beta}}$, respectively. Fig. 3 clearly shows how the heating efficiency rapidly decreases with diminishing $\hat{\beta}$ because of the decreasing compression ratio. As at low $\hat{\beta}(\lesssim 0.3)$ the plasma radius r_s strongly increases with rising λ[15, 16] the efficiency falls down at the lower right-hand corner of the figure.

Comparing the compression temperatures predicted by Eq. (5) with the temperatures measured in some pinches of high $\beta(\approx 0.7)$, i.e., the 145 kJ screw pinch at Garching[17] and the 1.1 MJ linear 8 m

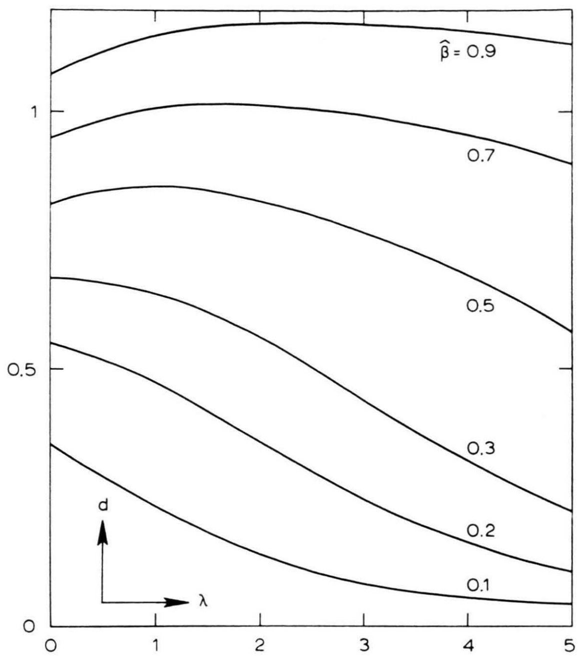

Fig. 3. Compression heating efficiency as a function of $\hat{\beta}$ and λ.

theta pinch at Culham[18], we find that the computed temperatures are too low by about 25 to 30%. The difference can be attributed to the neglect of ohmic electron heating, of which the contribution to the total temperature can be estimated[15] to be of this order if $\beta \approx 1$, while for $\beta \ll 1$ ohmic heating is negligible. In the derivation of Eq. (5) energy and particle losses were neglected, which could enlarge the difference. However, in the above-mentioned experiments these losses seem to be negligible.

We may note that in the case of an antiparallel trapped bias field Eq. (5) may give completely wrong results. Then the heating rate can be much higher because of the strong dissipation of the reverse trapped field[6, 7, 19, 20].

Further, the validity of Eq. (5) breaks down if the pressure is so low that effects of anomalous resistivity set in. Though in this region an enhanced electron heating occurs[21], the anomalous field diffusion leads to a broadening of the current sheath and therefore to a less effective ion shock heating[11, 21]. The net effect can result in a decreased

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total plasma temperature as compared with that predicted by classical theory. From the data given in Refs.[11, 18, 21] we have estimated that the anomalous effects occur if $p \lesssim 30\,\varrho/r_1^2$, where ϱ is the maximum ratio of the mean ion and electron energies during the fast shock; e.g., for $\varrho = 10$, $r_1 = 6$ cm if $p \lesssim 8$ mTorr.

3. Plasma Temperature in Dependence of $\hat{\beta}$ and Circuit Parameters

Equation (5) does not depend on the specific form of the field B as a function of time t. The derivative $\dot{B}$ is responsible for the dynamical shock heating, whereas the amplitude $\hat{B}$ gives the effect of the adiabatic heating. For a given capacitor energy storage optimum heating is achieved if $d(\dot{B}\hat{B})^{2/3}$ is made as large as possible. A method[3] to do this is to add to the main capacitor bank (1), which produces the confining field with amplitude $\hat{B}_1$, a small fast bank (2), which produces a fast rising field with a high value of $\dot{B}_2$. The combination of banks gives a better implosion heating. Bank (2) is switched first and after the fast compression the main bank (1) is fired (cf. Fig. 4).

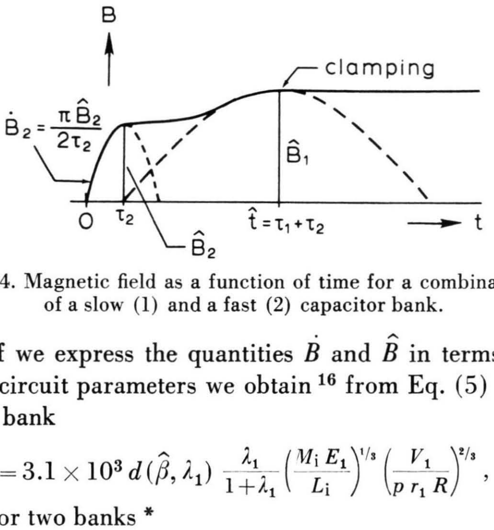

Fig. 4. Magnetic field as a function of time for a combination of a slow (1) and a fast (2) capacitor bank.

If we express the quantities $\dot{B}$ and $\hat{B}$ in terms of the circuit parameters we obtain[16] from Eq. (5) for one bank

$$k\hat{T} = 3.1 \times 10^3\, d(\hat{\beta}, \lambda_1)\, \frac{\lambda_1}{1+\lambda_1} \left(\frac{M_i E_1}{L_i}\right)^{1/3} \left(\frac{V_1}{p\, r_1 R}\right)^{2/3}, \quad (6)$$

or for two banks *

$$k\hat{T} = 3.1 \times 10^3\, d(\hat{\beta}, \lambda_2) \left(\frac{\lambda_1}{1+\lambda_1}\right)^{1/3} \left(\frac{\lambda_2}{1+\lambda_2}\right)^{2/3} \cdot \left(\frac{M_i E_1}{L_i}\right)^{1/3} \left(\frac{V_2}{p\, r_1 R}\right)^{2/3}, \quad (7)$$

* Equation (7) holds if the two banks are switched parallel as is the case in Ref. 3. If the banks are joined in series the gain in temperature will be even higher because then V_2 should be replaced by $V_1 + V_2$ in Eq. (7).

where the temperature, $k\hat{T}$, is expressed in eV. The factor $d(\hat{\beta}, \lambda_1)$ can be taken from Fig. 3. Further, E_1 is the energy of the main bank (1) in kJ; V_1, V_2 are the loading voltages (in kV) of banks (1) and (2), respectively (note that $\dot{B} \propto V$, $\hat{B} \propto E^{1/2}$); p is the filling pressure in mTorr; M_i is the ion mass in terms of the mass of a hydrogen atom; r_1 is the inner minor radius (in cm) and R is the major radius (in cm) of the toroidal discharge tube. Finally, L_i is the internal inductance (in nH), i.e., the coil inductance inside the tube (in MKS units $L_i = \mu_0 r_1^2 n^2/2R$, where n is the number of turns).

4. Numerical Results and Discussion

From Eqs. (6) and (7) it can be seen that for a given choice of the coil optimum heating is achieved in the case of high β if the external inductances are as low as possible: $\lambda_1 = \infty$ (one bank); $\lambda_1 = \lambda_2 = \infty$ (two banks). For a low-β plasma there exists an optimum value for the inductance parameter of the bank that determines the initial rise of the magnetic field, e.g., for $\hat{\beta} = 0.2$: $\lambda_1 = 1.5$ (one bank); $\lambda_1 = \infty$, $\lambda_2 = 1.25$ (two banks). The choice of the λ-values appears not to be critical as long as these are between about 1 and 2.

As an example, we take $\lambda_1 = 2$, $\lambda_2 = 1.5$, $p = 10$ mTorr, and $M_i = 2$ (deuterium); the properties of the chosen capacitor banks are given in Table 1. We consider two tori, one of standard dimensions and with a two turns-coil ($n = 2$) and a "fat" torus with a single-turn coil ($n = 1$). The calculated temperatures (in eV) are given in Table 1 for $\hat{\beta} = 0.2$ and 0.9.

Main bank	1000 kJ	20 kV	5000 μF	$\tau_{1/4} = 20\ \mu$s
Additional bank	72 kJ	60 kV	40 μF	$\tau_{1/4} = 2\ \mu$s

Torus 1	$R = 36$ cm	$r_1 = 6$ cm	$n = 2$	$L_i = 25$ nH
Torus 2	$R = 18$ cm	$r_1 = 8$ cm	$n = 1$	$L_i = 22$ nH

		one bank	two banks
Torus 1	$\hat{\beta} = 0.2$	142 eV	322 eV
	$\hat{\beta} = 0.9$	460	885
Torus 2	$\hat{\beta} = 0.2$	194	437
	$\hat{\beta} = 0.9$	628	1205

Table 1. Compression temperatures for various parameters.

The gain in temperature caused by the addition of the fast bank to the main bank is a factor of $2.25\,(\hat{\beta} = 0.2)$ or $1.92\,(\hat{\beta} = 0.9)$. If for the λ's the optimum values are chosen it follows from Eqs. (6) and (7) that the gain in temperature is about $(V_2/V_1)^{2/3}$.

In conclusion, we have shown how the compression temperature of a theta pinch scales with the β of the plasma and with the circuit parameters, and we have discussed a method to increase the implosion heating by the use of two capacitor banks rather than one.

The author thanks Professor C. M. BRAAMS for his interest, Dr. P. C. T. VAN DER LAAN and Dr. R. F. DE VRIES for many discussions, and Dr. C. BOBELDIJK for reading the manuscript.

This work was performed as part of the research programme of the association agreement of Euratom and the "Stichting voor Fundamenteel Onderzoek der Materie" (FOM) with financial support from the "Nederlandse Organisatie voor Zuiver-Wetenschappelijk Onderzoek" (ZWO) and Euratom.

Heating of Laser Produced Plasmas Generated at Plane Solid Targets

I. Theory

H. PUELL

Physik-Department der Technischen Universität München

(Z. Naturforsch. **25 a**, 1807—1815 [1970]; received 25 September 1970)

A hydrodynamic model for the heating of a plasma generated by the interaction of an intense giant laser pulse with a plane, solid target is developed. It is shown that in the case of nanosecond light pulses and a finite focal spot diameter, the plasma production may be considered as a steady state problem. Expressions for the electron temperature, the expansion energy of the ions, and the total particle number in the plasma as a function of the incoming light intensity are derived. An estimate of the ion temperature is discussed.

It has been shown in a number of publications [1] that during the interaction of intense laser light with solid materials a very dense and energetic plasma is formed. Representative values for the particle density and the electron temperature are 10^{21} cm^{-3} and several 100 eV [2] respectively. The kinetic energy of the ions measured at great distances from the target are in the range of several keV, depending on the target material [3].

In this paper we present a model for the heating of a laser produced plasma which is capable of explaining the difference between temperature and expansion energy noticed in previous experiments. We will restrict our considerations to the case of plane, solid targets. In the case of spherical targets we refer to papers by DAWSON [4] and FADER [5].

I. Ionization and Absorption

The transition of a solid material to a plasma under the influence of a strong radiation field is fairly well understood. In general, it is assumed that, first, some free electrons are created within the solid due to multiphoton transitions [6, 7]. These electrons are then multiplied by a cascade ionization process [8]. At light intensities of 10^{12} W/cm^2 a (nearly) complete ionization of the irradiated solid in a time $< 10^{-10}$ sec is achieved [1].

Therefore, for pulse durations $> 10^{-9}$ sec we may neglect the time necessary for sublimation and ionization in the following problem.

The heating of the plasma is due to energy absorption of the electrons from the radiation field

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